

Speed of sound in string gas cosmology

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ABSTRACT: We consider an ensemble of closed strings in a compact space with stable one cycles and compute the speed of sound resulting from string thermodynamics. Possible applications to the issue of Jeans instability in string gas cosmology are mentioned.

KEYWORDS: Cosmology of Theories beyond the SM, String theory and cosmic strings.

Contents

1. Introduction	1
2. Thermodynamics of string gas	3
3. Speed of sound of a string gas	4
4. Discussion and conclusions	7

1. Introduction

String gas cosmology [1] (see [2] for some recent reviews) is a scenario of the very early universe based on taking into account new degrees of freedom and new symmetries which characterize string theory but are absent in point particle field theories. In string gas cosmology (SGC), matter is treated as a gas of closed strings. According to SGC, the radiation phase of standard cosmology was preceded not by a period of inflation, but by a quasi-static Hagedorn phase during which the temperature of the string gas hovers close to the Hagedorn temperature [3], the maximal temperature of a gas of closed strings in thermal equilibrium. Thus, it is hoped that the scenario will not have a singularity in its past. SGC provides a possible dynamical explanation [1] of why there are only three large spatial dimensions (see, however, [4, 5] for some concerns), provides a simple and physical mechanism of stabilizing all of the size [6] (see also [7, 8]) and shape [9] moduli of the extra dimensions, leaving only the dilaton multiplet un-fixed. The dilaton, in turn, can be fixed [10] by making use of non-perturbative effects like gaugino condensation.

It has recently been suggested [11–13] that thermal fluctuations of a gas of closed strings on a toroidal space will, in the context of the background cosmology of the string gas scenario, generate an almost scale-invariant spectrum of adiabatic, coherent cosmological fluctuations, fluctuations which have all of the correct properties to explain the recent high precision observations of cosmic microwave background anisotropies. Thermal fluctuations are given by the specific heat capacity. For a string gas living in a toroidal space the heat capacity depends on the area of torus. The deviation from extensivity is, in fact, a result of the enormous number of winding modes that become excited close to Hagedorn temperature. This result suggests that the heat capacity $c_V(R)$ scales holographically as a function of the radius R of a volume embedded inside the total box, i.e. $c_V(R) \sim R^2$. This leads to the scale-invariance of the spectrum of cosmological fluctuations [11–13], and implements a concrete realization of the argument that holography almost always leads to scale invariance of perturbations [14]. SGC, in fact, produces a scale invariant spectrum of scalar metric perturbations with a slight red tilt, again like what is obtained

in most inflationary models. The spectrum of gravitational waves, on the other hand, is characterized [15] by a spectrum with a slight blue tilt, unlike the slight red tilt which is predicted in inflationary models. This yields a way to observationally distinguish between the predictions of SGC and inflation [16].

In order to develop string gas cosmology into a viable alternative to inflationary cosmology, further issues need to be addressed. How are the horizon, flatness, size and entropy problems of Standard Big Bang cosmology addressed in string gas cosmology? What explains the overall isotropy of the cosmic microwave background (horizon problem)? Why is the universe so large (size problem) and contains so much entropy (entropy problem) compared to what would be expected if the universe emerges from the Big Bang with a scale commensurate with its initial temperature? Recall that it was these questions which motivated the development of inflationary cosmology [17].

If SGC is embedded in a bouncing universe cosmology, as can be realized [18] in the ghost-free higher derivative gravity theory discussed in [19], then the horizon, size and entropy problems do not arise.¹ The flatness problem, however, persists. In this Note we focus on the flatness problem. The flatness problem has two aspects: firstly, what explains the overall nearly spatially flat geometry of the current universe, and, secondly, why there are no large amplitude small-scale fluctuations which will collapse into black holes and prevent the SGC scenario from working. It is this second aspect of the flatness problem which will be addressed in this Note (the first one is again not present if the Hagedorn phase of SGC occurs around the time of a cosmological bounce and we assume that at some point in the contracting phase the universe was as large as it is today and the spatial curvature is comparable to the current spatial curvature).

In cosmology, the Jeans length determines whether small scale structures can collapse. The Jeans length, in turn, is determined by the speed of sound. If the speed of sound is of the order unity (in units in which the speed of light is unity), then the Jeans length, the length below which perturbations are supported against collapse by pressure, is given by the Hubble length H^{-1} , where H is the expansion rate of space (see e.g. [21] for a review of the theory of cosmological fluctuations and [22] for an overview). In a radiation-dominated universe the speed of sound is $c_s^2 = 1/3$ and hence small-scale instabilities do not occur. In contrast, in a matter-dominated universe $c_s = 0$ and Jeans instability on small scales occurs. It is this gravitational instability which leads to the formation of nonlinear structures in the present universe.

In this Note we compute the speed of sound in the Hagedorn phase of SGC. Matter in the Hagedorn phase is dominated by relativistic strings containing both momentum and winding modes. On this basis, we might expect that locally flat space is protected against the Jeans instability. On the other hand, the net pressure is small since the positive pressure of the momentum modes is cancelled by the negative pressure of the winding modes. This leads to the expectation that the speed of sound might be small.

In this Note we show that the speed of sound calculated using the microcanonical ensemble is very small and positive. As soon as the radius of torus becomes two orders of

¹See [20] for an approach to resolving the size problem without assuming a bouncing cosmology.

magnitude or so larger than the string length, the speed of sound goes to zero exponentially with the radius. On this basis, it appears that SGC may suffer from a small-scale Jeans instability problem, since, in order to make contact with the current universe, the radius R needs to be at least 1 mm (if the string scale is comparable to the Grand Unification scale). However, since the microcanonical ensemble has limited applicability in describing the physics of subspaces of the entire space, we need other techniques in order to unambiguously resolve the Jeans instability puzzle of SGC.

There has been some previous work on inhomogeneities in the early phases of SGC. Specifically, in the context of taking the background of string gas cosmology to be described by dilaton gravity, as in [23–25], it has been shown [26] that there is no growth of cosmological inhomogeneities. Dilaton gravity cannot, as is now realized [27, 28], provide a consistent background for the Hagedorn phase of string gas cosmology because of the rapid time variation of the dilaton. Since the dilaton velocity was playing an important role in the considerations of [26], we have to revisit the issue of stability towards growth of fluctuations.

In the following section, we will review the microcanonical approach to string thermodynamics. We discuss the physics of an ideal gas of strings, the equilibrium conditions and the energy distribution in the gas. In section 3 we will focus on the microcanonical ensemble to find the speed of sound for string gas fluctuations. In the absence of a well-defined canonical ensemble, the interpretation of the speed of sound becomes subtle. The final section contains a discussion of the interpretation of results and conclusions.

2. Thermodynamics of string gas

There are two independent approaches to thermodynamics of string gas. The microcanonical ensemble approach starts with single string density of states calculated from the tower of states of a free string. Using this, one can find the number distribution of strings as a function of their energy. In this approach one finds the entropy of the microcanonical ensemble assuming equi-partition [29]. The second approach starts with Boltzmann equations and derives the equilibrium distribution of strings using the canonical ensemble [30]. The main advantage of the second approach is that it can study the time dependent properties of string gas. However, the canonical ensemble breaks down as the temperature reaches the Hagedorn temperature, rendering this approach useless for our purpose. Consequently, here we will make use of the first approach.

In spite of the fact that the canonical ensemble is not well-defined, one can still make sense of the partition function $Z(\beta)$ as a mathematical function, namely as the Laplace transform of the density of states $\Omega(E)$:

$$Z(\beta) = \int_0^\infty dE e^{-\beta E} \Omega(E) \quad (2.1)$$

where β is the inverse temperature. In the microcanonical ensemble approach, one employs the knowledge of the string spectrum to construct the partition function. Then, the density of states can be found by taking the inverse Laplace transform. The partition

function has singularities as values corresponding to distinct temperatures larger than the Hagedorn temperature. The position of the singularities of the partition function depends on the radius of compactification. As the torus grows in size, the singularities move closer to the Hagedorn temperature which we denote by $\beta = \beta_0$ [29]. The contribution of each singularity in the partition function to the density of states becomes smaller the further it is from the Hagedorn temperature.

As in previous work on SGC, we will work with the density of states Ω for a gas of strings in nine spatial dimensions, of which three are large (but compact). The reason for choosing compact rather than non-compact topology for our three large spatial dimensions is that string thermodynamics is better defined than in the non-compact case. If space is not compact, one does not obtain the holographic scaling of the specific heat capacity which is required to obtain a scale-invariant spectrum of cosmological perturbations. Also, it has been shown that in the context of a string gas in thermal equilibrium in three or more non-compact dimensions, at high energy densities a single energetic string captures most of the energy in the gas. More problematically, the specific heat turns out to be negative in this topology [29].

String thermodynamics tells us how the energy is being distributed among strings. If one defines $D(\epsilon, E)$ to be the average number of strings that carry energy ϵ while the whole gas has overall energy E , the energy distribution of the string gas in three compact but large dimensions follows figure 1 [29]. The absence of a peak which would correspond to the most probable value is the reason that assigning a mean value to a sub-box as we do in the canonical ensemble is not valid anymore. This means that although one can still calculate the partition function for a string gas, the canonical ensemble loses its ability to determine the thermodynamical quantities of a sub-box of the total box. Therefore, we have to restrict ourselves to the microcanonical ensemble. The physical way of understanding this is to notice that the thermodynamics of a string gas starts deviating from that of a gas of point particles as soon as there is enough energy to excite winding modes.² Winding modes are not localized degrees of freedom and result in non-extensivity of the entropy function (3.5). Non-extensivity implies that erecting a wall anywhere in the box will cause bulk effects, unlike in the case of a gas of point particles where it is just a surface effect [29].

In the next section we will calculate all the thermodynamical parameters that can be found using the microcanonical ensemble. Namely we will find the pressure, the temperature and the speed of sound for a gas of strings in a nine dimensional torus.

3. Speed of sound of a string gas

We will start with three large dimensions as a three-dimensional torus with equal radii R .³ As has been derived in [29], the density of states for fixed total energy E and fixed R and

²The thermodynamics of a gas of open strings is extensive and resembles that of a gas of point particles due to the absence of winding modes.

³What is essential for the analysis is the presence of one cycles on the manifold which guarantees the existence of stable winding modes.

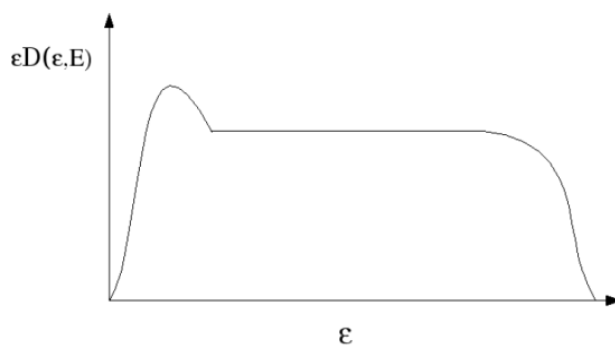


Figure 1: Energy distribution for a string gas in three large compact dimensions.

for energy densities above the Hagedorn energy density can be written as

$$\Omega(E, R) \simeq \beta_0 e^{\beta_0 E + a R^3} [1 + \delta\Omega_1], \quad (3.1)$$

where

$$\delta\Omega_1 = -\frac{(\beta_0 E)^5}{5!} e^{-(\beta_0 - \beta_1)(E - \rho_0 R^3)} \quad (3.2)$$

In string length units, β_0 the inverse Hagedorn temperature is simply 4π and β_1 is a constant smaller than β_0 which corresponds to the location of next singularity in the partition function [29] and $\rho_0 \sim l_s^{-4}$. In the above, a is constant (of the order l_s^{-3}) due to the fact that the momentum density is almost constant above the Hagedorn energy density.

The reason that we are interested in energy densities beyond Hagedorn energy density is that for lower values of energy density a gas of point particles becomes entropically favored and string gas acts effectively as a gas of low energy supergravity fields due to the lack of energy for exciting high oscillatory and winding modes. The ratio of the energy densities of a gas of closed strings to that of a gas of point particles is given by

$$\frac{S_{\text{strings}}}{S_{\text{point-particles}}} \sim \frac{\beta_0 \rho}{\rho^{d/(d+1)}} \sim \left(\frac{\rho}{\rho_0}\right)^{1/4} \quad (3.3)$$

It can be shown that for large dimensions $R \gg l_s$ [29]

$$\beta_0 - \beta_1 \sim \frac{l_s^3}{R^2}, \quad (3.4)$$

Since the entropy is the logarithm of the density of states, we obtain

$$S(E, R) = \ln \beta_0 + \beta_0 E + a R^3 + \ln(1 + \delta\Omega_1). \quad (3.5)$$

Knowing the entropy as a function of energy and radius we immediately obtain the following formula for the temperature

$$\begin{aligned} T(E, R) &= \left[\left(\frac{\partial S}{\partial E} \right)_V \right]^{-1} \\ &= \left[\beta_0 + \frac{1}{1 + \delta\Omega_1} \frac{\partial \delta\Omega_1}{\partial E} \right]_V^{-1}. \end{aligned} \quad (3.6)$$

Taking the derivative of (3.2) and keeping in mind that $\rho \gg \rho_0$ we get

$$\frac{\partial \delta \Omega_1}{\partial E} = -(\beta_0 - \beta_1) \delta \Omega_1. \quad (3.7)$$

Inserting this result into (3.6) and making use of (3.4) we obtain

$$\delta \Omega_1 = (\beta_0 - \beta) \frac{R^2}{l_s^3}. \quad (3.8)$$

For simplicity we define the large positive parameter

$$\tau = l_s (\beta - \beta_0)^{-1} \quad (3.9)$$

which is a measure of how close the system is to Hagedorn phase transition.

Inserting (3.8) into (3.2) and taking the logarithm of both sides results in

$$E \sim \frac{R^2}{l_s^3} \left[\ln \tau + 13 \ln \frac{R}{l_s} + 5 \ln(l_s^{-4} \rho) + \ln \frac{(\beta_0/l_s)^5}{5!} \right]. \quad (3.10)$$

The consistency of having $\delta \Omega_1 \ll 1$ implies, using (3.8), that in large radius limit $R \gg l_s$ we have $\tau \gg (R/l_s)^2$. Thus, for the approximations made in the current analysis of string gas thermodynamics in three compact large dimensions to be consistent, the temperature has to be extremely close to the Hagedorn temperature. In fact, for $R \gg 100l_s$, the first term in (3.10) dominates, resulting in a more severe constraint on the temperature, namely $\tau \sim \exp(\rho R l_s^3)$.

From our expression for the entropy, the pressure p for adiabatic perturbations can be found by taking the derivative with respect to volume at constant energy and making use of (3.8) to substitute for $\delta \Omega_1$. The result is

$$\begin{aligned} p &= T \left(\frac{\partial S}{\partial V} \right)_E \\ &= \frac{a}{\beta_0} - \left[a/\beta_0 + (2\rho + \rho_0)/3 \right] \tau^{-1} + O(\tau^{-2}) \end{aligned} \quad (3.11)$$

Note that the pressure close to Hagedorn phase transition has a maximum value of aT_0 at the Hagedorn temperature and declines very slowly with decreasing temperature. For energy densities much larger than the Hagedorn temperature, the magnitude of the pressure is negligible in comparison to the magnitude of the energy density. This result is expected since the positive pressure of the momentum string modes is cancelled by the negative pressure of the winding modes. It is the string oscillatory modes which are responsible for the net positive.

Taking the partial derivative of (3.5) with respect to the energy density at constant entropy one finds

$$\left(\frac{\partial R}{\partial \rho} \right)_S = -\frac{\beta_0 R}{3(\beta_0 \rho + a)} + O(\tau^{-1}) \quad (3.12)$$

Taking the derivative of (3.10) and inserting (3.12) therefore yields the following expression for the change of τ as one increases the density at constant entropy:

$$\left(\frac{\partial\tau}{\partial\rho}\right)_S = \left[Rl_s^3 - \frac{5}{\rho} - \left(l_s^3\rho - \frac{13}{R} \right) \frac{\beta_0 R}{3(\beta_0\rho + a)} \right] \tau + O(1) \quad (3.13)$$

The sound speed of the string gas can now be found by taking the derivative of (3.11) with respect to energy density at constant entropy and inserting (3.13). The result is

$$\begin{aligned} c_s^2 &= \left(\frac{\partial p}{\partial\rho}\right)_S \\ &= \left[Rl_s^3 - \frac{5}{\rho} - \frac{(l_s^3\rho - \frac{13}{R})\beta_0 R}{3(\beta_0\rho + a)} \right] \left[\frac{a}{\beta_0} + \frac{2\rho + \rho_0}{3} \right] \frac{1}{\tau} - \frac{2}{3} \frac{1}{\tau} + O(\tau^{-2}) \end{aligned} \quad (3.14)$$

The most important things to learn from the above result are firstly that for large enough values of R the speed of propagation of fluctuations in the string gas background in a box of radius R at temperature T and energy density ρ is positive, and secondly that it is extremely small in magnitude. The suppression of the magnitude is related to the fact that the pressure is suppressed relative to the energy density. In summary, as our most important results we have shown that

$$0 < c_s^2 = \frac{4}{9} \rho R l_s^2 (\beta - \beta_0) \ll 1. \quad (3.15)$$

In order for the approximations made in our derivation to be valid, the temperature has to be close to the Hagedorn phase transition temperature with an accuracy better than one part in $\exp(\rho R l_s^3)$. Thus, the speed of sound has the same behavior as it would in a very thin medium.

One can also calculate the speed of sound when the radius is kept constant instead of the entropy. The answer turns out to have the same $\rho R \tau^{-1}$ suppression.

$$0, < c_R^2 = \frac{2\rho R l_s^2}{3} (\beta - \beta_0) \ll 1, \quad (3.16)$$

which confirms the analogy with a very thin medium.

4. Discussion and conclusions

In this note we have computed the speed of sound in a gas of closed strings on a three dimensional torus, assuming that the toroidal radii are equal. Our main result is that the speed of sound is positive but highly suppressed when $R \gg l_s$.

The positivity of the speed of sound has implications for the stability of cosmological perturbations. In general relativity, for adiabatic fluctuations, the scalar metric perturbation variable Φ (the fluctuation of the metric component g_{00} in longitudinal gauge - the coordinate system in which the metric is diagonal - whose physical meaning is that of the relativistic generalization of the Newtonian gravitational potential) satisfies the following equation of motion (see e.g. [21] for a review of the theory of cosmological perturbations)

$$\Phi'' + 3\mathcal{H}\Phi' - c_s^2 \nabla^2 \Phi + [2\mathcal{H}' + (1 + 3c_s^2)\mathcal{H}^2] \Phi = 0. \quad (4.1)$$

In the above, we have used conformal time η which is related to the physical time variable t via the cosmological scale factor $a(t)$.

$$dt = a(t)d\eta. \tag{4.2}$$

The derivative with respect to conformal time is denoted by a prime, and \mathcal{H} is the scale factor in conformal time. Neglecting the expansion of the universe, the solutions of (4.1) are oscillatory if the speed of sound is positive:

$$\Phi_k(\eta) = Ae^{ikc_s\eta} + Be^{-ikc_s\eta}. \tag{4.3}$$

In particular, there is no instability of the metric fluctuations.

On small scales, however, we should be concerned with the instability of density fluctuations. On sub-Hubble scales and in a matter-dominated universe, the density fluctuation $\delta\rho$ obeys the following equation of motion

$$\ddot{\delta\rho} - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = 0, \tag{4.4}$$

once again neglecting both the expansion of space and the possible presence of entropy fluctuations. This equation shows that fluctuations on scales larger than the Jeans length (given by the wave number for which the second and the third term in the above are equal) grow. If the speed of sound is comparable to the speed of light, the Jeans length is of the order of the Hubble length H^{-1} and there is no Jeans instability problem. However, if the speed of sound is negligible (as in our case) then there is a potential Jeans instability problem.⁴

Since in the Hagedorn phase of string gas cosmology p/ρ is vanishingly small for large values of R , the equation of state is like that of a matter-dominated universe. However, since the basic objects which make up the gas are not point particles but extended relativistically moving strings, it is unlikely that equation (4.4) applies to describe matter fluctuations in SGC. Putting these considerations together, we conclude that our study has so far not resolved the concern that SGC might suffer from a Jeans instability problem.

The interpretation of speed of sound in SGC is somewhat subtle. The reason is that in the thermodynamics of a string gas the equivalence of the microcanonical and canonical ensembles is lost due to the exponentially growing density of states close to the Hagedorn temperature. Thus, results concerning the change in thermodynamical quantities as the size of the entire sample box is changed cannot immediately be applied to questions related to sub-boxes, like the question we are addressing here.

In standard thermodynamics the analogy between the canonical ensemble and the microcanonical ensemble in the limit of large box sizes comes about because we can approximate the partition function — the Laplace transform of the density of states — by a saddle-point approximation, finding the average value of E for any sub-system of interest

⁴Note that the expression for the speed of sound (3.15) is independent of the string coupling g_s . The string thermodynamics is defined in the limit of small g_s the same way as in [32]. The Jeans length can be found using $R \geq \sqrt{\frac{\pi c_s}{g_s^2 \rho}}$.

with small fluctuations about the mean. However, if the density of states grows exponentially with energy, like in the case of strings, the saddle point approximation breaks down. If one tries to push the canonical ensemble further, one obtains divergent fluctuations about the mean value.

Although non-extensive thermodynamics might seem very counter-intuitive, even classically in the presence of gravity it is somewhat inevitable. According to the ergodicity theorem, a system that evolves for a long time will be able to reach any small neighborhood of a point in phase space. Therefore, there are trajectories which run into regions in phase space that correspond to black holes. Once these black holes are nucleated out of thermal fluctuations, they grow and render the canonical ensemble ill-defined.

Acknowledgments

This work is supported in part by an NSERC Discovery Grant, by the Canada Research Chair program, and by a FQRNT Team Grant. We wish to thank Andrew Frey for valuable conversations.

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